Introduction and Background

In fluid mechanics, a fluid passing through an orifice constriction experience a drop in pressure across the orifice. With the time recorded for the required level in the volumetric tank to rise from 10 liters to 20 liters and the diameter measured, the pressure drop can then be used to calculate the flow rate of the fluid. This experiment allows us to detect the effect of friction on water flow. In determining the performance of water through a jet and the effect of friction on performance, the coefficient of discharge (C_d), the coefficient of velocity (C_v) and coefficient of contracta (C_c) are needed. These three coefficients allow us to understand the effect of the friction on water flow more clearly. Each of this coefficient is a ratio of the actual performance to ideal performance as related to discharge, velocity, and contraction respectively. All of the three coefficients are also a measure of pressure loss. From this experiment, it is evident that the energy loss in the system can be directly related to the difference in head levels between ideal and actual conditions.

The objective of this lab is to use the Orifice and Jet Apparatus; following up with the application of the Bernoulli's equation to the data collected. From this, the coefficient of velocity (C_v), contraction (C_c), and discharge (C_d) of a small orifice can be obtained.

Theory

Exercise 1: Determination of Coefficient of Velocity from Jet Trajectory

The velocity of the ideal orifice outflow at the narrowest diameter (vena contracta) is

$$v_i = \sqrt{2 * g * h}$$

Where h, is the height of the fluid above the orifice. The true velocity of the orifice outflow is

$$v = C_v\sqrt{2gh} \quad (1)$$

Where C_v is the coefficient of velocity because this allows the horizontal component of the jet is assumed to be constant if air resistance is neglected, so that at the time, t, the horizontal distance covered is

$$x = v * t \tag{2}$$

The vertical component, however, is affected by gravity throughout the same time, t, so the vertical distance covered is defined as

$$y = g * \left(\frac{t^2}{2}\right) \tag{3}$$

which can be manipulated to give

$$t = \sqrt{2 * \left(\frac{y}{g}\right)} \tag{4}$$

Combining (3) and (2) and (1) into (2) gives

$$C_v = \frac{x}{2\sqrt{y*h}} \tag{5}$$

Based on these equations given a constant h value (steady flow), C_v can be calculated by the jet x, y coordinates. Plotting x by $\sqrt{y * h}$ will give a value of $2C_v$ for its slope.

Exercise 2: Determination of Coefficient of Discharge under Constant Head

In this exercise, the coefficient of discharge is determined. Utilizing Bernoulli's Equation again, the actual flow rate of the water jet is

$$Q_t = A_c v \tag{6}$$

A_c is the cross-sectional area of the narrowest diameter and is defined as

$$A_c = C_c A_o \tag{7}$$

C_c in this equation is the coefficient of contraction, and A_o is the area of the orifice. Therefore, $Cc < 1$, and (6) can be rewritten as

$$Q_t = C_c * A_o * C_v * \sqrt{2 * g * h} \tag{8}$$

C_d is equal to the product of C_v & C_c and is known as the discharge coefficient, so (8) takes the form

$$Q_t = C_d * A_o * \sqrt{2 * g * h} \tag{9}$$

Plotting a graph of Q_t against $\sqrt{h}$ gives us a linear function, and so the slope, S, of that line will be $S = C_d * A_o * \sqrt{2 * g}$ only if C_d is assumed to be constant. Thus, manipulating this equation the value of C_d can be expressed as

$$C_d = \frac{slope}{A_o\sqrt{2g}} \tag{10}$$

Part 2: The Bernoulli's Theorem Apparatus

Bernoulli's Theorem states that: "The total head of flowing liquid between two points remains constant provided there is no loss due to friction and no gain due to the application of outside work between the two points".

Thus, for two different points 1 and 2

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} \tag{11}$$

If point 1 and point 2 are of different diameters, then V_1 and V_2 are different comparing total head at any point along the Venturi tube equating the total head.

$$Q = A_1V_1 = A_2V_2 \tag{12}$$

Putting the value of V_1 from equation (12) in equation (11)

$$\frac{P_1}{\rho g} + \frac{{V_2}^2}{2g}\left(\frac{A_1}{A_2}\right)^2 = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} \tag{13}$$

Also referring to pressure change

$$Q = C^*A_2\sqrt{\frac{2g(H_1-H_2)}{1-\frac{{A^2}_2}{A_1}}} \tag{14}$$

Where C is known as the coefficient of the meter that can be established by an experiment.

Equipment and materials

Stop Watch: Used to measure the amount of time elapsed from starting time to stop time.

H1D Volumetric Bench: Provides a controlled flow of water used to measure the volumetric flow rate.

F1-17 Orifice and Jet Apparatus: Shoots a trajectory of water under a certain flow rate used to measure the position of the jet of fluid.

Figure 1: Stop Watch (retrieved from (2))

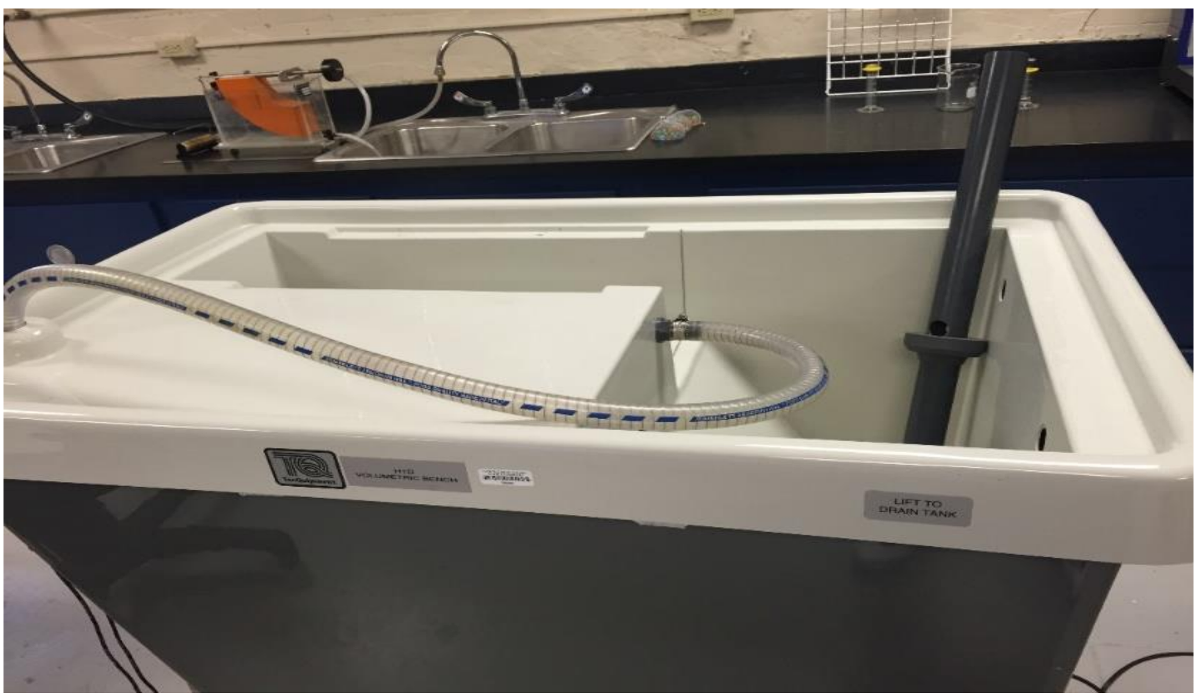

Figure 2: Volumetric Bench (retrieved from (1))

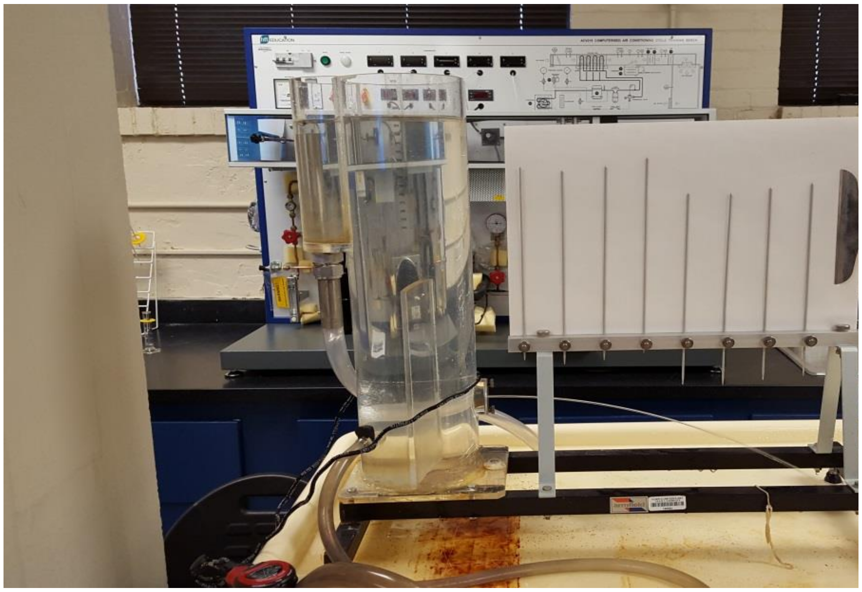

Figure 3: F1-17 Orifice and Jet Apparatus (retrieved from (1))

Procedure

Exercise 1: Finding Coefficient of Velocity from Jet Trajectory

First, secure a sheet of paper on the backboard with the clamp and assure that its upper edge is horizontal. Then, adjust the overflow tube to give a high head and note the value of the head. Once the jet was turned on and steady flow was established, the needles were then adjusted to mimic the shape of the trajectory of the jet stream. The needle points were positioned just above the stream. Dashes were then made right above the tops of the needles on the sheet of paper. Note the horizontal distance from the plane of the orifice (deemed as the x = 0 points) should be close enough to the orifice to treat it as having the value y = 0. From this point the y displacements were measured, given that the downward y-direction is positive. Once all of the measurements have been made, repeat this procedure for a low reservoir head. Finally, plot x vs $\sqrt{y * h}$ and determine the slope of the graph. From this graph, find the velocity coefficient C_v which is equal to the average slope/2.

Exercise 2: Finding the Coefficient of Discharge Under Constant Head

First, adjust the overflow tube to the upper position (higher) head. After the steady was achieved, the flow rate was measured by timing how long it took to fill up from one designated point in the tub area to another specified point. The reservoir head value was noted. Repeat this procedure for different head values.

Part 2: The Bernoulli's Theorem Apparatus

First, the hydraulic bench is connected to the test set, with the outlet valve open. Secondly, the pump started and the bench outlet valve was slowly opened to obtain a small flow. Next, the test set outlet valve was adjusted so that water levels in all manometer tubes could be observed.

Closing /opening of outlet valves was done to raise or lower the water levels. After moving the total head close to the tube midpoint, the flow rate and manometer readings were recorded.

Discussion of Results

The coefficient of velocities C_v for two different flow rate was found at 0.536 and 0.506. These values are less than 1 which verifies the theoretical value.

The coefficient of discharge C_d is a property from the orifice, not from the water, and it would depend on the pressure difference on both sides of the orifice. So, it is justifiable to assume that C_d is constant over that range. C_d is the ratio of the of the from the experimental discharge to the ideal/theoretical discharge and is equal to the C_v multiplied to the C_c. Thus, the value of C_d is less than 1.0.

The Bernoulli's theorem test verifies the theory of total head of flowing liquid between two points remains constant. The coefficient of the meter C is also determined from the collected data. This value is dependent on the value of flow rate. It was observed that the higher the flow rate, higher the value of C.

Conclusions

This experiment helps us in understanding how to measure experimentally the coefficient of velocity (C_v), the coefficient of discharge (C_d) and the coefficient of a meter (C). Though these values were in acceptable range some errors could have occurred from stopwatch timing the flow rate. If more trials are taken the data will be more accurate.

References

1. Miller, R. W. (1983). *Flow Measurement Engineering Handbook.*

2. Baker, R. C. (2005). *Flow measurement Handbook: industrial designs, operating principles, performance, and applications.* Cambridge University Press